



Sigma exchange in the nuclear force and effective field theory

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Received 14 April 2006; received in revised form 11 August 2006; accepted 12 October 2006

Available online 7 November 2006

Editor: J-P. Blaizot

Abstract

In the phenomenological description of the nuclear interaction an important role is traditionally played by the exchange of a scalar $I = 0$ meson, the sigma, of mass 500–600 MeV, which however is not seen clearly in the particle spectrum and which has a very ambiguous status in QCD. I show that a remarkably simple and reasonably controlled combination of ingredients can reproduce the features of this part of the nuclear force. The use of chiral perturbation theory calculations for two pion exchange supplemented by the Omnes function for pion rescattering suffices to reproduce the magnitude and shape of the exchange of a supposed σ particle. I also attempt to relate this description to the contact interaction that enters more modern descriptions of the internucleon interaction.

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When describing QCD to non-physicists, we generally say that it is the theory that accounts for nuclear binding. However, in practice our understanding of the precise way that QCD leads to nuclear bound states is still not good. Nuclear binding is most commonly described by an internucleon potential which can be parameterized by the exchange of mesons [1–3]. A key feature is that there is an attractive component to the central potential with an intermediate range. This component is often parameterized by the exchange of a scalar isoscalar meson, the sigma, of mass around 500–600 MeV. While other exchanges in the potential are correlated with clear resonances seen in the particle spectrum, the sigma is a puzzle. It is not seen in the usual way in the spectrum and, after 40 years of debate, does not have a clear interpretation in terms of the quarks and gluons of QCD.¹ It is unfortunate that this ingredient in the signature effect of the strong interactions has such an ambiguous status.

The expectation is that the sigma represents, in some way, the exchange of two pions. The quantum numbers certainly are correct for this. Sophisticated attempts that construct the po-

tential from scattering data (e.g. [6]) have two pions as the lightest intermediate state. However, while phenomenologically useful, these are not able to answer the question of the fundamental nature of the central interaction. Modern descriptions of the internucleon interaction use chiral perturbation theory to calculate two pion exchange at low energy [7–11]. This description appears to provide a good description of the longest range part of the internucleon force and can be used to describe this component of nucleon–nucleon scattering. However, the chiral amplitude does not produce a potential in agreement with that expected for sigma exchange. As shown in Fig. 8 of Ref. [8], the resulting potential grows too strong at moderate distances. This problem is readily traceable to the fact that the chiral amplitudes grow monotonically with the energy and hence get very large at moderate energies. In this Letter I add a simple and well-motivated addition to the chiral description, i.e. the Omnes function describing pion rescattering. We will see that this will produce an interaction remarkably close in structure to the exchange of a 600 MeV sigma meson. It is clear that a sigma resonance is not the driving feature of this calculation, yet the needed properties of sigma exchange are reproduced.²

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¹ A discussion of the status of the sigma which is very much in the spirit of the present work can be found in [4]. A careful recent analysis of $\pi\pi$ scattering describing the sigma as a pole on second sheet, quite far from the real axis, is found in [5].

² A few other attempts to describe the nuclear interaction without a sigma are seen in [12].

Recently, there have been successful applications of ideas of effective field theory in which the nuclear interaction is treated not by potentials but by contact interactions—delta function interactions [7,13]. At low energy (recall that the energy typical of nuclear binding is 10 MeV/nucleon) the result of the exchange of a heavy particle can be described by a local interaction. Mathematically, this is consistent with the potential description because, as the mass m gets large, the Yukawa potential forms a representation of a delta function. Physically, this follows from the uncertainty principle, as the exchange of a heavy particle has a short range. Nonlocality, to the extent it is needed, can then be described by contact derivative interactions. This development greatly increases the generality of the description of nuclei, as it reduces multiple potentials with different functional forms to a small number of constants giving the strengths of the contact interactions. After discussing the potential treatment, I will also attempt to make contact with this effective field theory description.

As the primary tool, I will use the dispersion relation derived by Cottingham, Vinh Mau and others [6]. For textbook reviews, see [1,2]. The scattering amplitude for two nucleons obeys an unsubtracted t-channel dispersion relation.

$$M(s, t) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} d\mu^2 \frac{\text{Im } M(s, \mu^2)}{\mu^2 - t - i\epsilon}. \quad (1)$$

The imaginary part of this amplitude is connected to the on-shell amplitudes in the crossed channel $N\bar{N} \rightarrow N\bar{N}$ with the important intermediate state being that of two pions. The overall amplitude is decomposed into partial waves described by their spin and isospin quantum numbers. The greatest interest in this Letter will be on the scalar–isoscalar ($J = 0, I = 0$) channel. By taking the nonrelativistic limit and ignoring the energy dependence in the S channel, one can define a momentum space potential that depends only on the momentum transfer q^2 . For the scalar isoscalar central potential, let us define the corresponding spectral function

$$\rho_S(\mu) = \text{Im } M_S(q = i\mu). \quad (2)$$

In terms of this imaginary part the potential is defined by the dispersion relation

$$V_S(q^2) = \frac{2}{\pi} \int_{2m_\pi}^{\infty} d\mu \mu \frac{\rho_S(\mu)}{\mu^2 + q^2},$$

$$V_S(r) = \frac{1}{2\pi^2 r} \int_{2m_\pi}^{\infty} d\mu \mu e^{-\mu r} \rho_S(\mu). \quad (3)$$

Because ρ_S describes physical on-shell intermediate states, this formalism provides a well defined tool either for the analysis of nucleon scattering or for theoretical attempts to describe the nuclear interaction. Much recent work (see [8–11] and references therein) has used this formalism to match to chiral descriptions.

At low energy these spectral functions can be rigorously calculated in chiral perturbation theory. The imaginary part of the Feynman diagrams describe the physical intermediate

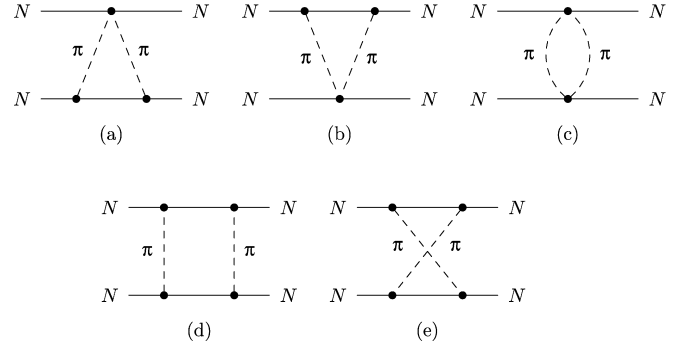


Fig. 1. Two pion exchange diagrams which arise in chiral perturbation theory.

states and generate the spectral function ρ_S . These can be calculated either through the direct calculation of the Feynman diagram, or by appropriately multiplying together the relevant on-shell $\pi N \rightarrow \pi N$ scattering diagrams [14]. For the diagrams of Fig. 1 a, b, c these imaginary parts are [8,9]

$$\rho_S^{a,b}(\mu) = \frac{3g_A^2}{64F_\pi^4} [4c_1 m_\pi^2 + c_3(\mu^2 - 2m_\pi^2)] \times \frac{(\mu^2 - 2m_\pi^2)}{\mu} \theta(\mu - 2m_\pi), \quad (4)$$

$$\rho_S^c(\mu) = -\frac{3}{32\pi F_\pi^4} \sqrt{1 - \frac{4m_\pi^2}{\mu^2}} \theta(\mu - 2m_\pi) \times \left(\left[4c_1 m_\pi^2 + \frac{c_2}{6}(\mu^2 - 4m_\pi^2) + c_3(\mu^2 - 2m_\pi^2) \right]^2 + \frac{c_2^2}{45}(\mu^2 - 4m_\pi^2)^2 \right). \quad (5)$$

Here c_1, c_2, c_3 are parameters that describe the $NN\pi\pi$ vertex—these have been measured in pion nucleon interactions [8,9,11,15,16]. I will address the box and crossed box diagrams below. These spectral functions are valid in the low energy regime only, and one observes that they grow monotonically with the energy.

However, there is another ingredient which necessarily enters. In the description of the $\pi\pi$ system, unitarity requires the inclusion of $\pi\pi$ rescattering. For a single elastic partial wave, unitarity of the S matrix and analyticity require a unique form of the solution, given originally by Omnes [17]. The amplitudes in the elastic region are described by a polynomial in the energy times the Omnes function

$$\Omega(\mu) = \exp \left[\frac{\mu^2}{\pi} \int \frac{ds}{s} \frac{\delta(s)}{s - \mu^2} \right]. \quad (6)$$

Here δ is the $\pi\pi$ scattering phase shift, in our case for the $I = 0, J = 0$ channel. Chiral perturbation theory is consistent with this order by order in the energy expansion. Following Ref. [18], it is known how to match this general description to the results of chiral perturbation theory by appropriately identifying the polynomial. The elastic region in this channel extends effectively up to energies of 1000 MeV.

In practice there has been good success at using the lowest order chiral amplitudes, supplemented by the Omnes function.

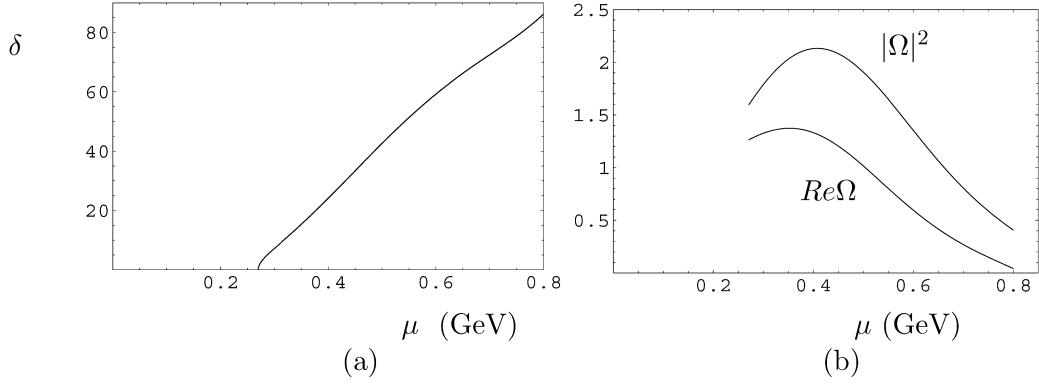


Fig. 2. The left graph gives the pion phaseshifts that are the input into the Omnes function, while the right figure shows the real part and the absolute square of the Omnes function.

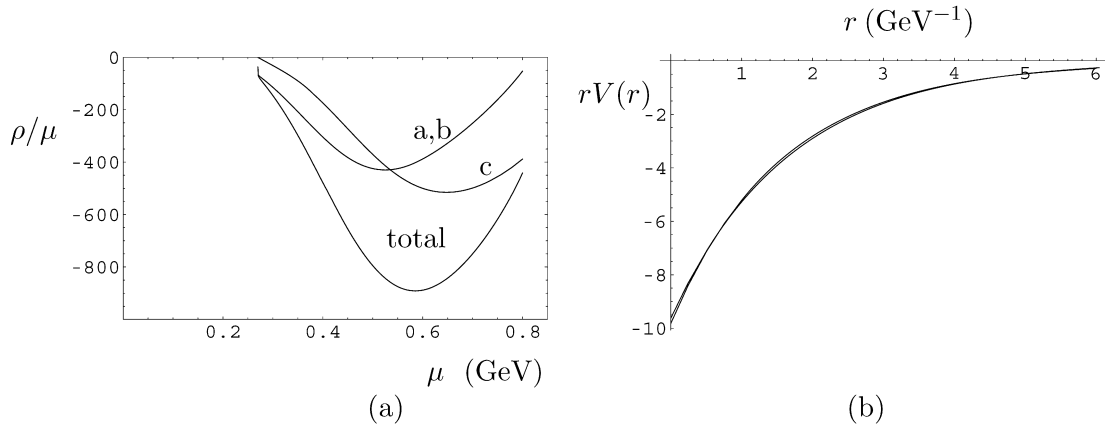


Fig. 3. The left figure shows the our results for the spectral function $\rho(\mu)/\mu$ as well as the individual components of diagram 1 a, b, c. The right figure shows the coordinate space potential $rV(r)$. There are actually two curves in the figure on the right. One is the result of this calculation and the second is that of a narrow 600 MeV sigma, with normalization chosen to match. The curves cannot be distinguished.

An example which is close to the present problem is $\gamma\gamma \rightarrow \pi\pi$ in the S wave. Here the lowest order calculation results in an amplitude which also grows monotonically and which violates unitarity near 600 MeV [19]. However, the addition of the Omnes function [20] tames this runaway growth. When the Omnes function and the lowest order amplitude are combined the result is in close agreement with both experiment and with a two loop chiral calculation up to energies beyond 700 MeV [21]. This procedure is equally rigorous as the usual chiral method at low energies. At higher energies, it can be adapted order by order. The Omnes function captures some of the features that would emerge if chiral perturbation theory were applied at higher order—it captures a subset of diagrams that relate to unitarization. While a full description clearly requires a complete set of higher order calculations, the Omnes method can be useful in those cases where pion rescattering is strong. In practice, it is most important when the $\pi\pi$ system is in an S-wave.

I will adopt the Omnes solution matched to the leading order chiral result, and will explore possible modifications below. The description of the spectral function then becomes

$$\rho_S(\mu) = \rho_S^{a,b} \text{Re } \Omega(\mu) + \rho_S^c |\Omega(\mu)|^2. \quad (7)$$

The phase shifts can be analyzed in chiral perturbation theory in combination with experiment, with the definitive treat-

ment of Colangelo et al. (CGL) [22]. Their result for the $I = 0$, $J = 0$ phase shift is shown in Fig. 2, along with the resulting Omnes functions.³ Note that there is no sigma resonance visible in the phase shift near 500–600 MeV. A resonance in the elastic region is manifest by the phase shift passing through 90 degrees, which certainly does not happen near the sigma mass. If one explores the complex plane there is a pole on the second sheet very far from the real axis [5]. However, the resonance is not the driving force in the description of the $\pi\pi$ amplitude at these energies. Instead, the chiral amplitude can be parameterized by a few low energy constants, which in turn are more determined by the $\rho(770)$ than by the sigma [23].

With these ingredients, we can display the result for the scalar interaction. In Fig. 3, I show the result for ρ , along with the individual contributions of the diagrams of Fig. 1. If we had a pure sigma exchange this would be a delta function at the mass of the σ , or a Breit–Wigner shape corresponding to a narrow resonance. One could be forgiven for seeing this result

³ In producing the Omnes function, I had to extend the phase shifts above the $\mu = 850$ MeV endpoint of the CGL analysis in order that the principle value part of the Omnes function integral be well behaved near the upper end. I have explored several smooth extensions, with residual effects at the few percent level.

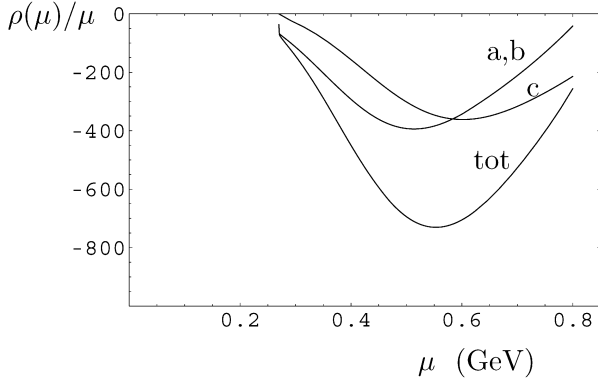


Fig. 4. The spectral function $\rho(\mu)/\mu$ in the presence of higher order energy dependent modifications as described in the text.

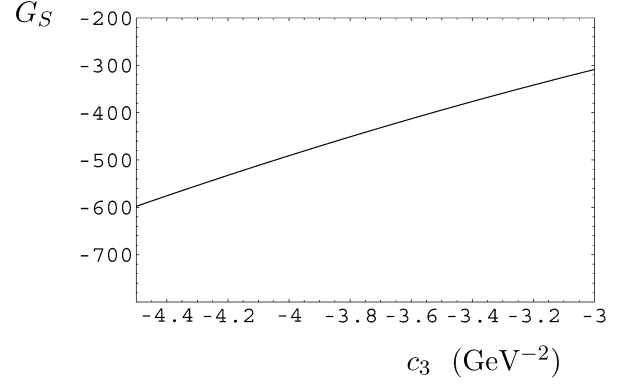


Fig. 5. The strength of the scalar interaction as a function of the parameter c_3 .

as a very broad resonance, even though no resonance exists in the formalism. The coordinate space potential is also shown in Fig. 3. Also shown for comparison is the potential of an infinitely narrow 600 MeV scalar with a normalization chosen to match. In practice these are hard to differentiate because the curves are nearly identical. The simple description of Eq. (5) reproduces closely the spatial variation of the sigma potential. This says that these results have a range which is capable of the describing the intermediate range attraction in the central potential which is needed for nuclear binding. The strength of the interaction will be addressed below.

One can address the robustness of this result by considering possible higher order modifications of the basic representation. The $NN\pi\pi$ interaction has been described by the lowest order chiral Lagrangian. There are also energy dependent modifications to these low order results. In particular, when studying on-shell vertices such as those that go into the spectral function we most often find form factors that depend on the energy. It would be reasonable to expect that the $NN\pi\pi$ interaction would be modified by a form factor such as

$$c_3 \rightarrow \frac{c_3}{(\mu^2 + m^2)^n}. \quad (8)$$

Indeed, there is theoretical expectation that this should occur. A study of the ingredients of this coupling [24] suggests that 55–70% of c_3 arises from integrating out Δ exchange in the t-channel. The Δ propagator then leads to an energy dependence of the vertex. Following the techniques of [14], it is straightforward to incorporate the Δ propagator into these diagrams.⁴ I have explored the effects of form factors and the Δ propagator. While these influence the magnitude appreciably, as described below, there is not a qualitative change in the shape of the spectral function. For example, taking the extreme case that all of c_3 arises through Δ exchange in the t-channel, the modification of spectral functions is shown in Fig. 4. While the relative contribution of the two types of diagram changes slightly, the energy variation and spatial variation are remarkably similar to

the original case. The use of a pure monopole or a dipole form factor does not change this conclusion.

Let me address the strength of the interaction by considering the integral of the spectral function through the energy region under consideration,

$$G_s = \frac{2}{\pi} \int_{2m_\pi}^{0.8 \text{ GeV}} \frac{d\mu}{\mu} \rho_s(\mu). \quad (9)$$

This is just the integral of the curves shown in Figs. 3a, 4. In potential models with the exchange of a narrow sigma, this has the value

$$G_s^\sigma = \frac{g_\sigma^2}{m_\sigma^2}, \quad (10)$$

which numerically is often given as $G_s^\sigma = 300\text{--}450 \text{ MeV}^{-2}$ [3,13]. The result depends most sensitively on the parameter c_3 , which is not perfectly known. The phenomenological extraction of c_3 from πN and NN data has a large error bar, $c_3 = -4.7^{+1.2}_{-1.0} \text{ GeV}^{-2}$ [8,11,15,16]. However, when using an Omnes representation, it is likely that this constraint is on the product $c_3 \Omega(2m_\pi)$, in which case the value would be $c_3 = -3.7^{+1.0}_{-0.8} \text{ GeV}^{-2}$. (The other parameter choices used were $c_1 = -0.64 \text{ GeV}^{-2}$ and $c_2 = 3.3 \text{ GeV}^{-2}$, although these have only a small impact on the results.) The result for G_s as a function of c_3 is shown in Fig. 5. There is rough agreement for the required range of magnitudes of G_s for the allowed values of c_3 [7]. Here the use of a form factor does make a difference. With the inclusion of the effect of the Δ propagator, the result is 20% smaller. If we use a straight monopole formfactor with a mass $m = 800 \text{ MeV}$ the value of G_s is 40% smaller than without it for a given value of c_3 . These examples show the model dependence of the higher order effects. At present understanding the differences may be accounted for by adjusting the value of c_3 . These uncertainties in the appropriate values of c_3 and G_s keep us from using the magnitude of the potential as a precise test of the method.

Now consider the effective field theory description of the interaction. In a Wilsonian effective field theory treatment, one treats the light degrees of freedom (pions in this case) dynamically up to a scale Λ . This means that we consider tree and loop diagrams with energies below this scale. Physics beyond this

⁴ The formulas of Section III of [14] apply directly with the modification $\tau = (t + 2m_\Delta^2 - 2m_N^2 - 2m_\pi^2)/2\sqrt{t - 4m_\pi^2} \xi m_N$ whenever there is a Δ propagator.

scale is treated by a contact interaction, i.e. $G_s(\Lambda)$ —a local interaction that parameterized the residual physics from energies above Λ . Epelbaum, Glöckle and Meißner (EGM) [10] have implemented such a treatment using the chiral amplitudes without the Omnes function in the spectral function with an energy cutoff. The ideal use of the present calculation would be to use the Omnes description in a treatment like EGM. The Omnes description appears to be a better representation of the long distance physics than just using the bare chiral amplitudes. There is no loss of rigor in adding the Omnes function to the chiral predictions at low energy as long as the matching is done correctly. In addition, the Omnes function tames the runaway growth of the bare chiral amplitudes. However, at higher energies, the present procedure is clearly not rigorous and there is inherently some uncertainty in the dynamical calculation, as we have demonstrated above. The contact interaction arising from the chiral Lagrangian would serve to correct any flaw in the dynamical calculation in order to fully agree with reality. To the extent that this dynamical calculation is a good one, the residual contact interaction would be small. Indeed, from the agreement seen above for the rough magnitude and shape of the potential, it plausible that the residual contact interaction could be negligible compared to the primary dynamical effect of two pion exchange.⁵

However, the EGM approach is not the standard effective field theory treatment. More commonly, the usual pion exchange diagrams, Fig. 1d, e, are treated dynamically, but those of Fig. 1a, b, c are not explicitly considered. This is the case when dimensional regularization is used rather than an energy cutoff, because the former diagrams are finite and the latter are divergent when treated at any given order in the energy expansion. The effects of these diagrams are then included in the contact interaction. This contact interaction is given by the momentum space potential at zero momentum

$$G_s^{\text{eff}} = V(q=0) = \frac{2}{\pi} \int_{2m_\pi}^{\infty} \frac{d\mu}{\mu} \rho_S(\mu). \quad (11)$$

To the extent that significant contributions do not come from energies above the end of our calculation (800 MeV), our calculation then provides an estimate of the strength of the contact interaction in the scalar-isoscalar channel.

In order to assess this result it is perhaps easiest to compare with Ref. [26] (EMGE). These authors have considered the extraction of the effective field theory coefficients from the data, including the effects of regularizing the calculations. They have also compared to the phenomenology of integrating over the potential or integrating out bosonic resonances. For potentials with a sigma effect, their results for this channel amounts to $G_s^{\text{eff}} = g_\sigma^2/m_\sigma^2$ plus smaller corrections from higher scalar resonances. While again the rather large uncertainties in the magnitude of the present calculation make it any precise conclusions possible, it then appears that the magnitude discussed above is also roughly appropriate for the effective field theory

description. Note that present effective field theory treatments can differ in how the contact interaction is treated. In some applications, such as [13], the contact interaction can be directly used without further modifications. In others, such as [7], a smearing or renormalization of the contact interaction is implemented to deal with divergences in that calculation. In such situations, the appropriate renormalized value may be different and a matching of that calculation to the spectral description would be needed to compare the values.⁶

Potentials can have different meanings in different contexts [27], and in different calculational schemes there can be different values of the contact interaction [28]. Therefore let me specify more fully the scheme of the present calculation. In the chiral treatment, one keeps pion exchange as an explicit degree of freedom while treating the shorter range interactions as contact terms. The $NN\pi$ vertex is the one in the usual baryon chiral Lagrangian. In such a treatment, we should treat the box and crossed-box diagrams of Fig. 1 dynamically, and they should therefore not be included into the contact interaction. For this reason I did not include these diagrams in the calculation of the integrand $\rho_S(\mu)/\mu$ whose integral gives the strength of the contact interaction. In frameworks other than the one considered here, it might be appropriate to include some of the box and/or crossed-box diagrams into the description of the contact interaction. However, it seems that for the scalar central potential the iteration of the one pion interaction is a numerically small compared to the irreducible two pion/sigma contribution, for example see [8] and Fig. 3.15 of Ref. [2]. With slight modifications, then, it appears that the present calculation could also be adapted to other frameworks.

Chiral perturbation theory plus the Omnes function give a quite simple description of the scalar central potential, with a result very similar to the exchange of a conventional sigma particle. However the physics important in this calculation is not a sigma resonance, but rather only chiral amplitudes and the Omnes function. This description appears to be robust, being qualitatively unchanged by the addition of higher order interactions. Besides elucidating a long standing puzzle, these results are useful because we have a reasonable control over all the main ingredients, the chiral amplitudes and the $\pi\pi$ phase shifts. The connection of the nuclear interaction to QCD becomes more under control.

Acknowledgements

I would like to thank especially Thibault Damour for many conversations during the course of this work, U. von Kolck for discussions about the iterated one pion interaction, R. Furnstahl and B. Serot for information on the nuclear interaction and J. Gasser, Barry Holstein and H. Leutwyler for continually informative discussions on the dispersion representation and the Omnes function. I also appreciate the hospitality the

⁵ This is further discussed in [25].

⁶ Experience from the meson sector [23] suggests that similar evaluations of chiral coefficients of that sector match well to dimensionally regularized coefficients with scales of 500–700 MeV.

IHES where most of this work was performed. This work has been partially supported by the US National Science Foundation.

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